

# A PHYSICAL MODEL FOR NUCLEATE BOILING HEAT TRANSFER

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**Abstract**—In this paper an equation for the heat-transfer coefficient from a horizontal surface to a boiling liquid is established starting from a physical model for the nucleation and hydrodynamical processes.

## NOMENCLATURE

<p><math>a</math>, thermal diffusivity;</p> <p><math>b</math>, exponent;</p> <p><math>C_D</math> drag coefficient;</p> <p><math>c</math>, specific heat of the liquid;</p> <p><math>D</math>, diameter of a bubble;</p> <p><math>D_o = 2R_o</math>;</p> <p><math>g</math>, gravitational acceleration;</p> <p><math>J</math>, mechanical equivalent of heat;</p> <p><math>l</math>, size of a gas filled cavity;</p> <p><math>\bar{l}</math>, average value of the sizes of the gas-filled cavities;</p> <p><math>L</math>, average distance between successive active centres;</p> <p><math>L_o</math>, average distance between successive gas-filled cavities;</p> <p><math>m, m^I, m^{II}, m^{III}, m^{IV}</math>, exponents;</p> <p><math>n</math>, number of active centres per unit area;</p> <p><math>N</math>, number of gas-filled cavities per unit area;</p> <p><math>p</math>, pressure;</p> <p><math>\Delta p</math>, supersaturation of the liquid;</p> <p><math>q</math>, heat transferred per unit area in unit time;</p> <p><math>r</math>, latent heat of vaporization;</p> <p><math>R</math>, radius of the bubble;</p> <p><math>R_o</math>, radius of the bubble at the moment when the bubble breaks off;</p> <p><math>s</math>, length in Nusselt number;</p> <p><math>t</math>, time;</p> <p><math>T_l</math>, temperature in the bulk of the liquid (<math>^{\circ}\text{K}</math>);</p> <p><math>T_w</math>, temperature at the wall;</p>	<p><math>\Delta T = T_w - T_l</math>;</p> <p><math>u_b</math>, velocity in the resistance term of equation (26);</p> <p><math>\alpha_1</math>, average value of heat-transfer coefficient during the time interval <math>\tau_1</math>;</p> <p><math>\alpha_2</math>, average value of heat-transfer coefficient during the time interval <math>\tau_2</math>;</p> <p><math>\alpha_t</math>, average value of the heat-transfer coefficient on the surface <math>L^2</math> at the time <math>t</math>, measured from the moment when the nucleus begins to grow;</p> <p><math>\alpha</math>, heat-transfer coefficient given by equation (14);</p> <p><math>\gamma'</math>, specific gravity of the liquid;</p> <p><math>\gamma''</math>, specific gravity of the vapour;</p> <p><math>\eta</math>, dynamical viscosity;</p> <p><math>\phi</math>, angle of contact between bubble and wall at the moment when the bubble breaks off;</p> <p><math>\lambda</math>, thermal conductivity;</p> <p><math>\nu</math>, kinematic viscosity;</p> <p><math>\rho</math>, liquid density;</p> <p><math>\rho_c</math>, length given by equation (4);</p> <p><math>\tau_1</math>, the duration of bubble growth on an active centre;</p> <p><math>\tau_2</math>, time interval which elapses from the moment when a bubble breaks off until a new bubble starts growing on the same active centre;</p> <p><math>\tau = \tau_1 + \tau_2</math>;</p> <p><math>\sigma</math>, surface tension liquid-vapour;</p> <p><math>Nu = \frac{\alpha s}{\lambda}</math>;</p>
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$$Pr = \frac{\nu}{\alpha};$$

$Re$ , Reynolds number;

$$Re_1 = \frac{2R(dR/dt)}{\nu};$$

$$Re_2 = \frac{qR_o}{r\gamma''\nu}.$$

### INTRODUCTION

THE CORRELATION of the experimental results regarding heat transfer from a horizontal surface to a boiling liquid has been made in literature by deducing—from certain physical considerations—several dimensionless groups and by experimentally determining the functional dependence between them. In all these papers one considered that the rate of heat transfer is determined by the stirring action of the bubbles. The differences between the numerous correlations proposed [1–8] arise from the way in which are taken into account: (i) the hydrodynamic process of stirring of the liquid in the vicinity of the heating surface and, mainly, (ii) the nucleation process. The points of view of the different authors have been examined critically by Westwater [9] and Kutateladze [10].

However, the mechanisms proposed in the above mentioned papers do not permit the elaboration of a physical model which could serve for a quantitative formulation of the problem of heat transfer in the case of boiling. Some attempts to propose mechanisms which could permit such quantitative formulations have been made only recently in references [11–14]. However, while the models proposed by Zuber [12] and Tien [14] refer only to the hydrodynamic process, the model proposed in [11] and [13] contains a description of both the hydrodynamic and the nucleation process.

This paper presents a synthesis of the author's earlier results [11, 13].

### MECHANISM OF HEAT TRANSFER FROM A HORIZONTAL SURFACE

In the case of nucleate boiling, in certain privileged points of the heating surface (active centres) bubbles are formed continuously and break off as soon as they reach a certain volume.

The mixing caused by the bubbles traversing the liquid renders the temperature uniform throughout the bulk of the liquid, so that a sensible temperature variation exists only in the immediate vicinity of the heating surface. Consequently the rate of heat transfer is determined by the hydrodynamic process taking place in the liquid in the immediate vicinity of the heating surface. The greatest part of heat is transferred directly to the liquid which, owing to superheating, transfers it further on to the surface of the bubbles growing on the active centres or traversing the liquid. Recently [15, 16] it has been suggested that for large heat fluxes the rate of heat transfer is determined by the vaporization of a liquid microlayer at the base of the bubble into its interior. These cases will be treated in another paper. In the following we shall consider that the rate of heat transfer depends upon the motions taking place in the liquid at the immediate vicinity of the heating surface, these motions being caused by the growth of the bubbles on the active centres of the surface.

Let us therefore consider (a) the problem of the growth of a bubble on an active centre, and (b) the problem of the physical meaning of an active centre.

(a) A vapour bubble grows on an active centre according to an expression of the form:

$$R = \frac{(T_w - T_l) c\gamma' (\pi a)^{1/2} t^{1/2}}{r\gamma''}. \quad (1)$$

This equation has been established by Forster and Zuber [17] for a vapour bubble growing in a superheated liquid and may be used in a first approximation for a bubble growing on an active centre of the heating surface [18].

The bubble breaks off from the active centre as soon as its radius  $R$  becomes equal to  $R_o$ .

For the radius  $R_o$  of a bubble departing from an active centre the equation generally used is

$$R_o = 0.01 \vartheta \left( \frac{\sigma}{\gamma' - \gamma''} \right)^{1/2}. \quad (2)$$

Equation (2) has been established under the assumption that the bubble breaks off when the buoyant force and that due to surface tension become equal. Cole [19] has pointed out that at

large heat fluxes the resistance force of the liquid must be taken into account too, but we have not yet an equation for  $R_0$  which may be used in this case.

The duration  $\tau_1$  of bubble growth on an active centre may be estimated by means of equations (1) and (2)

$$\tau_1 = \frac{R_0^2}{\pi a} \left( \frac{r\gamma''}{(T_w - T_l)c\gamma'} \right)^2. \quad (3)$$

At the moment when the bubble breaks off, a new element of liquid comes into contact with the heating surface, and a certain time interval  $\tau_2$  elapses until a new bubble starts growing on the same active centre. From the experimental data of Jakob [1], obtained for relatively low heat fluxes, it follows that  $\tau_1$  and  $\tau_2$  are of the same order of magnitude. On the other hand, the experimental data of Westwater and Santangelo [20] confirmed by those obtained by Donald and Haslam [21] show that for large heat fluxes  $\tau_2 \ll \tau_1$ .

(b) From thermodynamic considerations one may draw the conclusion that the formation of growing vapour bubbles, in a liquid subjected to boiling, requires a certain superheating of the liquid as well as the presence of nuclei, i.e. of bubbles whose radius is larger or equal to  $\rho_c$  [1, 22, 23].

$$\rho_c = \frac{2\sigma\gamma'}{(\gamma' - \gamma'')\Delta p}. \quad (4)$$

Clausius-Clapeyron's equation allows equation (4) to be written

$$\rho_c = \frac{2\sigma T_l}{\gamma' r J \Delta T}. \quad (5)$$

The theory of nucleation in the homogeneous case\* shows that the rate of formation of the nuclei is small (even practically negligible) up to a certain critical value of the ratio  $(p + \Delta p)/p$  but becomes infinite for this critical value [24]. Therefore, if the nuclei would only appear owing to statistical fluctuations, there would follow that the liquid would boil gently until in the

vicinity of the heating surface it becomes sufficiently superheated for the ratio  $(p + \Delta p)/p$  to reach its critical value. However, once this value is reached, the rate of nucleation becomes infinitely high and for this reason the layer of liquid in the immediate vicinity of the wall is spontaneously converted into vapour. The rate of nucleation in the case of nucleate boiling does not depend on the superheating  $(T_w - T_l)$  as shown above but is much higher. This fact is due to the structure of the heating surface, as has been proved conclusively by Corty's and Foust's experiments [25]. These authors ascertained that the rougher the surface, the more intensive is the heat transfer in the case of boiling. This might be explained by the presence of gas-filled cavities (pits and scratches [26-28]) on the metal surfaces, which can, for certain values of superheating, play the part of nuclei. During boiling, the gas is eliminated, so that the continuation of boiling is rather difficult to explain in this manner. Corty [29], however, put forth the hypothesis, also sustained by Mesler and Banchemo [30], that each vapour bubble breaking off from the heating surface leaves behind it an extremely small bubble. The continuation of boiling could therefore, be explained possibly in this manner. In other words, the presence of gas-filled cavities on the metal surfaces initiates the boiling, but it continues owing to the small bubbles left by the large ones which break off.

If we assume that these cavities are of conic shape, we may say that such a gas-filled cavity becomes an active centre as soon as the superheating of the liquid is strong enough so that the size  $l$  of the gas-filled cavity may be given by:

$$l = \frac{2\sigma T_l}{(T_w - T_l)\gamma' r J}. \quad (6)$$

This conclusion has been arrived at in a number of papers [31-33] and is of fundamental importance in deducing an equation for the heat-transfer coefficient.

#### HEAT-TRANSFER COEFFICIENT

The process of heat transfer is characterized by a double periodicity: a temporal periodicity,†

\* In the heterogeneous case which is of interest in this paper, things do not generally stand otherwise, from a qualitative point of view, if the solid surface is "smooth".

† This temporal periodicity is not the same for each active centre.

at time intervals  $\tau = \tau_1 + \tau_2$  and a spatial quasi-periodicity, at distances  $L$  ( $L$  being the average distance between successive active centres).<sup>\*</sup> Therefore, the process taking place on a surface of area  $L^2$  around the active centre, in the time  $\tau$ , characterizes the process of heat transfer on the entire plate. It is therefore reasonable to define the heat-transfer coefficient as an average, both on the surface  $L^2$  and in the time  $\tau$ . (Obviously, for the time  $\tau$ , an average value must be considered.)

In the growing stage of the bubble on an active centre the stirring caused by this growth in a point situated within a surface of area  $L^2$  around the active centre considered, depends on the radius  $R$  of the bubble, on its rate of growth  $dR/dt$ , on the dynamic viscosity  $\eta$  of the liquid, on the density  $\rho$  of the liquid, on the distance to the active centre, on the distances from the point considered to the neighbouring active centres and on the radius of the bubbles and their rate of growth in these neighbouring centres. The dependance on the latter distances and lengths is due to the stirring caused at the point taken into consideration by the growth of the bubbles in the neighbouring active centres. For the sake of simplicity, it will be assumed that, at a certain moment, all the bubbles have the same radius. The heat-transfer coefficient at the point taken into consideration depends on the quantities mentioned above and on the thermal conductivity  $\lambda$  and the specific heat  $c$ . Dimensional considerations permit to write for the average value of the heat-transfer coefficient on the surface  $L^2$  at the time  $t$ , measured from the moment when the nucleus begins to grow<sup>†</sup>

$$\frac{\alpha_t R}{\lambda} = F \left[ \frac{2R (dR/dt)}{\nu}, \frac{\nu}{a'}, \frac{R}{L} \right]. \quad (7)$$

Equation (1) shows that  $R (dR/dt)$  is time independent, having the value

$$R \frac{dR}{dt} = \frac{1}{2} \left[ \frac{c\gamma' (\pi a)^{1/2} \Delta T}{r\gamma''} \right]^2.$$

<sup>\*</sup> A spatial "quasi"-periodicity is mentioned since the distance between the active centres is not the same.

<sup>†</sup> The form of the function  $F$  depends, too, on the geometric arrangement of the active centres on the heating surface.

The group  $[2R (dR/dt)]/\nu$  has the structure of a Reynolds number.

The average value  $\alpha_1$  of  $\alpha_t$  for the duration  $\tau_1$  of the growing stage of the bubble is given by the equation:

$$\alpha_1 = \frac{1}{\tau_1} \int_0^{\tau_1} \alpha_t dt. \quad (8)$$

Therefore,

$$\begin{aligned} \alpha_1 &= \frac{\lambda L}{\tau_1 R (dR/dt)} \int_0^{R_0/L} F \left[ \frac{2R (dR/dt)}{\nu}, \frac{\nu}{a'}, \frac{R}{L} \right] d \frac{R}{L} \\ &\equiv \frac{\lambda L}{\tau_1 R (dR/dt)} F_1 \left[ \frac{2R (dR/dt)}{\nu}, \frac{\nu}{a'}, \frac{R_0}{L} \right], \end{aligned} \quad (9)$$

wherefrom

$$\frac{\alpha_1 R_0}{\lambda} = F_2 \left[ \frac{2R (dR/dt)}{\nu}, \frac{\nu}{a'}, \frac{R_0}{L} \right]. \quad (10)$$

Some comments will now be made with respect to the ratio  $R_0/L$  which appears in equation (10). Let us denote  $N$  the number of gas-filled cavities per unit area. For a given superheating  $\Delta T$  of the liquid in the immediate vicinity of the heating surface, only those gas-filled cavities for which  $l \geq (2\sigma T_l)/(r\gamma''J\Delta T)$  are liable to become active. Denote by  $\varphi$  the distribution law of the cavities. In other words  $N\varphi dl$  represents the number of gas-filled cavities having  $l$  comprised between  $l$  and  $l + dl$ . The function  $\varphi$  is dependent on  $l$  and dimensional considerations make it certain that  $\varphi$  is dependent on  $l$  through the dimensionless ratio  $l/l_0$ ,  $l_0$  being a length whose significance will be discussed below. It is, generally speaking, hardly probable that  $l_0$  should be the only parameter appearing in the expression of  $\varphi$ . One must therefore consider that  $\varphi(l/l_0)$  only characterizes a certain surface.

The fraction of the number of cavities, which is active for a certain superheating, is therefore given by

$$\int_{\frac{2\sigma T_l}{r\gamma''J\Delta T}}^{\infty} \frac{\varphi(l/l_0) dl}{\int_0^{\infty} \varphi(l/l_0) dl} = \psi \left( \frac{2\sigma T_l}{l_0 r\gamma''J\Delta T} \right). \quad (11)$$

From the average value  $\bar{l}$  of  $l$

$$\bar{l} = \frac{\int_0^{\infty} l\varphi(l/l_0) dl}{\int_0^{\infty} \varphi(l/l_0) dl} = \text{const. } l_0$$

it is seen that  $l_0$  is proportional to  $\bar{l}$ .

For a random distribution of the active centres, the average distance  $L$  between successive active centres may be estimated by aid of the expression

$$L = N^{-1/2} \psi^{-1/2} = L_o \psi^{-1/2}.$$

Therefore,

$$\frac{L}{R_o} = \frac{L_o}{R_o} \psi^{-1/2}. \quad (12)$$

In equation (12) there appear two "lengths" which are characteristic for a given surface, namely: the average distance  $L_o$  between the gas-filled cavities of the surface and the average value  $l$  of the sizes of these cavities.

As pointed out above, it is possible (and even probable) that the function  $\varphi(l/l_o)$  (and therefore also the function  $\psi$ ) characterizes only a given surface.

Therefore, for a given surface, there results from equations (10) and (12)

$$\frac{\alpha_1 R_o}{\lambda} = F_2 \left[ \frac{2R (dR/dt)}{\nu}, \frac{\nu}{a}, \frac{R_o}{L_o} \psi^{1/2} \right]. \quad (13)$$

The heat-transfer coefficient may be defined by means of the equation:

$$\alpha = \frac{\alpha_1 \tau_1 + \alpha_2 \tau_2}{\tau}, \quad (14)$$

where  $\alpha_2$  is the average value of the heat-transfer coefficient during the time interval  $\tau_2$ .

Since, for usual values of heat fluxes  $\tau_2 \ll \tau_1$ , the heat-transfer coefficient  $\alpha_1$  corresponding to the growth period of the bubble coincides with the heat-transfer coefficient  $\alpha$ .

The group  $[2R (dR/dt)]/\nu$  which appears in equation (13) may be also written by using (1), (3) and  $a = q/(T_w - T_l)$  in the following equivalent forms:

$$\frac{2R (dR/dt)}{\nu} \equiv \frac{R_o^2}{\tau_1 \nu} \equiv \pi \frac{\nu}{a} \left( \frac{\alpha R_o}{\lambda} \right)^{-2} \left( \frac{q R_o}{r \gamma' \nu} \right)^2, \quad (15)$$

wherefrom one may draw the conclusion that [11]

$$\frac{\alpha R_o}{\lambda} = \tilde{f} \left( \frac{q R_o}{r \gamma' \nu}, \frac{\nu}{a}, \frac{L_o}{R_o}, \frac{\sigma T_l}{l_o r \gamma' J \Delta T} \right). \quad (16)$$

In order to establish the form of the function  $\tilde{f}$  a physical model for the hydrodynamical process is needed. Such a model will be suggested in the following section.

### A MODEL FOR THE HYDRODYNAMICAL PROCESS

Let us consider a group of active centres, with an average distance  $L$  between successive centres and let us assume for the sake of simplicity that simultaneously in each centre there appears a bubble nucleus. It is plausible to assume, that in the immediate vicinity of the heating surface the liquid flow, caused by the growth of the bubbles, is nearly parallel to this surface up to a distance  $L/2$  from the active centres, beyond which it becomes practically normal to the heating surface, the liquid mixing up with the bulk.

We shall assume that this motion is quasi-stationary and that at each instance along the distance  $(L/2) - R$  one may use for the heat-transfer coefficient the equation which is valid in stationary case for a semi-infinite fluid in laminar flow along a plate, the liquid velocity in the initial point being equal to the growth velocity  $dR/dt$  of the bubbles. A model somehow nearer to reality would correspond to a radial motion, on a plate, of a liquid which has its sources distributed on the lateral surface of a semi-infinite cylinder normal to the plate, the liquid velocity in the initial point being proportional to  $dR/dt$ . Such a model is at present under investigation; it will be published later on. The simplified representation used in this paper is a good approximation of the above model for all times for which  $L/2$  is not too large as compared with  $R$ .

With the assumptions mentioned above the following expression is obtained for the average of the heat-transfer coefficient over the distance  $(L/2) - R$ :

$$\alpha'_t = 0.68 \lambda \left\{ \frac{dR/dt}{\nu [(L/2) - R]} \right\}^{1/2} Pr^{1/3}. \quad (17)$$

Since only a fraction of the heating surface is in contact with the liquid, the remainder being covered with vapour bubbles, we may write, in first approximation, the following expression for the heat-transfer coefficient  $\alpha_t$ :

$$\alpha_t = 0.68 \lambda \left\{ \frac{dR/dt}{\nu [(L/2) - R]} \right\}^{1/2} Pr^{1/3} \left[ 1 - \pi \left( \frac{R}{L} \right)^2 \right]. \quad (18)$$

Therefore

$$\alpha = \frac{1}{\tau} \int_0^{\tau} 0.68\lambda \left[ 1 - \pi \left( \frac{R}{L} \right)^2 \right] \times \left\{ \frac{dR/dt}{\nu[(L/2) - R]} \right\}^{1/2} Pr^{1/3} dt. \quad (19)$$

In a first approximation we use for the radius  $R$ , equation (1).

If we replace  $dt$  in equation (19) by  $(dt/dR) dR$ , we obtain, after some transformations, the following final equation [13]

$$\frac{\alpha R_o}{\lambda} = 1.36 \frac{L}{R_o} \left[ \frac{R (dR/dt)}{\nu} \right]^{1/2} Pr^{1/3} \times \int_0^{R_o/L} \frac{(1 - \pi x^2)x^{1/2}}{(\frac{1}{2} - x)^{1/2}} dx \equiv 1.36 \left[ \frac{R (dR/dt)}{\nu} \right]^{1/2} Pr^{1/3} F_3 \left( \frac{R_o}{L} \right), \quad (20)$$

where\*

$$F_3 \left( \frac{R_o}{L} \right) \equiv \frac{L}{R_o} (0.399 - 0.127u_o - 0.05 \sin u_o + 0.036 \sin 2u_o - 0.016 \sin^3 u_o) \quad (21)$$

and

$$u_o \equiv \arccos \left( 4 \frac{R_o}{L} - 1 \right).$$

The function  $F_3$  is plotted in Fig. 1.

**DISCUSSION OF THE EQUATIONS**

The equations proposed in literature for the correlation of the experimental results may all be written in the general form

$$Nu = \text{const. } Re^m Pr^{m^1}.$$

The differences between them consist in different choices of the characteristic length and velocity in the expressions of Nusselt and of Reynolds number.

Forster and Zuber [5] use for Reynolds number the expression

$$Re_1 \equiv \frac{2R (dR/dt)}{\nu}$$

\* For integration the change of variable  $\cos u = 4x - 1$  has been made.

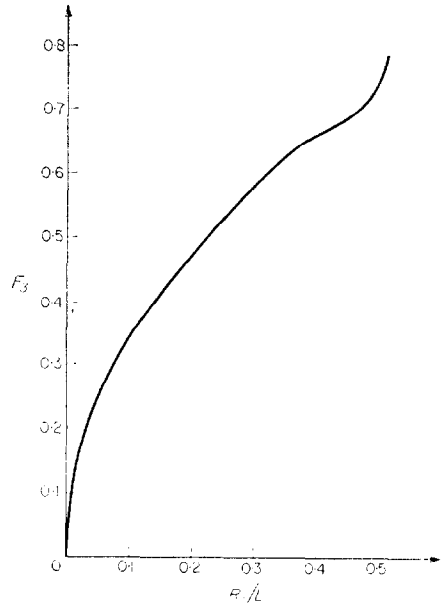


FIG. 1.

whilst other authors use the ratio†

$$Re_2 \equiv \frac{qR_o}{r\gamma''\nu}.$$

For the characteristic lengths which appear in Nusselt's number, Kruzhilin [2] uses the expression  $R_o (\rho_c/R_o)^{m^{11}}$ , Kutateladze [4] the expression

$$R_o \left\{ \frac{p}{[\sigma(\gamma' - \gamma'')]^{1/2}} \right\}^{m^{111}}$$

and Forster and Zuber [5] equation (1) in which  $t^{1/2}$  is replaced by  $(2\sigma/\Delta p)^{1/2} (\gamma'/g\Delta p)^{1/4}$ . We remark that the length  $s$  takes into account, at least partially, the nucleation process. The drawback of these correlations consists in the fact that the structure of the heating surface is not taken into account.

In another series of correlations one uses for  $s$  expressions of the form  $s = R_o(nR_o^2)^{m^{1v}}$ , where  $n$  represents the number of active centres

† We remark that using equation (3) one may easily show that  $Re_1$  and  $Re_2$  are related by:

$$Re_1 = \pi Pr \left( \frac{\alpha R_o}{\lambda} \right)^{-2} Re_2^2.$$

per unit area [8]. Through  $n$  the structure of the heating surface is taken into account this type of correlation cannot be used as long as the manner in which  $n$  depends on  $\Delta T$  is not known.

The equation proposed by the author [11] removes, at least partially, the deficiencies of the two types of correlations mentioned above. Equation (16) takes into account both the structure of the surface and the manner in which  $n$  depends on  $\Delta T$ .

A detailed comparison with experiment of the equations obtained is not yet possible, for the following reasons:

(a) the distribution law of cavities depths is not known;

(b) no information is available concerning the two "lengths"  $l_o$  and  $L_o$  which according to the theory developed here characterize the heating surface;

(c) we do not know an expression for  $R_o$  except for the limiting case of sufficiently small heat fluxes.

Moreover, even if all this information were available, it is not very likely that a quantitative agreement with experiment could be obtained, mainly, because of the numerous approximations made in deducing equation (20).

We notice however that the equations obtained seem to be in agreement with the experimental results obtained by Nishikawa *et al.* [34] and by Gaertner and Westwater [35]. Nishikawa *et al.* [34] found that for water boiling on a smooth horizontal brass surface,

$$a \sim n^{1/3}, \quad (22)$$

where  $n$  is the number of active centres per unit area. More recently Gaertner and Westwater have shown that

$$a \sim n^{0.43}. \quad (23)$$

If one supposes that  $R_o$  does not depend on  $\Delta T$ , equation (20) leads to

$$a \sim \Delta T F_3 \left( \frac{R_o}{L} \right).$$

If function  $F_3$  is written in the form  $F_3 \sim (R_o/L)^b$  one obtains for the exponent  $b$  a value close to  $\frac{1}{3}$ . Equation (20) leads therefore to a dependence of the form  $a \sim \Delta T n^{1/4}$ . Since  $n$  increases with

increasing  $\Delta T$ , it follows that equation (20) is not in disagreement with equations (22) and (23).

Equation (20) permits also, to a certain extent, to justify theoretically the following empirical equation, (24), proposed by Forster and Zuber for the heat-transfer coefficient at the maximum flux\*

$$\frac{a_s}{\lambda} = 0.0015 Re_1^{0.62} Pr^{1/3}. \quad (24)$$

Considering that at the maximum heat flux  $L \approx 2R_o$ , equation (20) leads to

$$\frac{a R_o}{\lambda} \sim Re_1^{1/2} Pr^{1/3}. \quad (25)$$

As suggested by Cole [19] at the maximum heat flux it is necessary to use for  $R_o$  the equation

$$(\gamma' - \gamma'') \frac{\pi D_o^3}{6} = \pi D_o \sigma f(\theta) + \frac{\pi D_o^2}{8} C_D \frac{\gamma'}{g} u_b^2, \quad (26)$$

where the first term on the right-hand side represents the force due to surface tension, while the second one represents the resistance force. Evaluating the velocity  $u_b$  by  $u_b \approx 2(dR/dt)$  and using for  $R$  equation (1), one obtains

$$(\gamma' - \gamma'') \frac{\pi D_o^3}{6} = \pi D_o \sigma f(\theta) + \frac{\pi}{2} C_D \frac{\gamma'}{g} \left[ \frac{(\Delta T) c \gamma' (\pi a)^{1/2}}{r \gamma''} \right]^4. \quad (27)$$

If we take into account the approximations implied in equation (27) it is clear that this equation can give only qualitative informations concerning  $R_o$ . Therefore a quantitative comparison of equation (25) with experiment is not yet possible. We notice however that the exponent of the Reynolds number in equation (25) has a value near to the experimental one and that in the characteristic length used in Nusselt's number the same quantities appear as in the experimental one.

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\* As characteristic length in Nusselt's number, Forster and Zuber use the expression (1), in which  $t^{1/2}$  is replaced by

$$(2\sigma)^{1/2} \gamma'^{1/4} g^{-1/4} \left[ \frac{J r \gamma' \gamma'' \Delta T}{T_i (\gamma' - \gamma'')} \right]^{-3/4}.$$

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**Résumé**—Dans cet article, synthèse de deux mémoires précédents, on établit une équation donnant le coefficient de transmission de chaleur à partir d'une surface horizontale, en utilisant un modèle physique des phénomènes hydrodynamique et de "nucléation".

**Zusammenfassung**—In dieser Arbeit, einer Synthese zweier früheren [11, 13], wurde eine Gleichung für den Wärmeübergangskoeffizienten von einer waagerechten Oberfläche aufgestellt, indem ein physikalisches Modell für die Blasenbildung und die hydrodynamischen Erscheinungen zugrundegelegt wurde.

**Аннотация**—В статье, являющейся обобщением двух предыдущих статей [11, 13], на основании физической модели процессов образования пузырьков и гидродинамики выводится уравнение коэффициента переноса тепла от горизонтальной поверхности.